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The Option to Re-organize Firms with Valuable Growth Options and Risky Bond Prices

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ABSTRACT We develop a structural model of corporate bond prices which accounts for the option held by bondholders to liquidate or re-organize a firm that ends up in bankruptcy. Although there is a wide body of literature in corporate finance that examines the trade-offs between liquidation and re-organization for creditors in financially distressed firms, our paper is the first to incorporate these options into the ex-ante value of risky debt. The option to liquidate or to re-organize has value in our model due to the presence of valuable growth options. Liquidation value being the depreciated value of assets in place, ignores the value of these options. But were bondholders to take over control of the firm, the value of the firm is, with some efficiency loss, the sum of the value of assets in place and the value of future growth options.

I. INTRODUCTION

Bondholders suffer a loss on their investment when a firm enters financial distress. They can minimize their loss by selecting the method of recovery of their outstanding claim. Bondholders can choose to liquidate the firm through a Chapter 7 filing. Or, they may choose to reorganize their debt. Reorganization can take the form of renegotiating their claim under Chapter 11 protection, or, through a recapitalization by swapping debt for equity. There is an extensive literature in corporate finance that examines how these bondholder options affect the cost of financial distress.[1] Yet, these options have not been incorporated into a structural model for corporate bond prices. This paper aims to fill this gap.

Merton (1974) established the now well-known result that the value of risky debt is the value of safe debt less the value of a put option on the firms' underlying assets. We extend this result to show that in addition to safe debt and a put option, the price of a risky bond also includes a warrant on the firm's assets. The terms of the warrant are such that exercise of the warrant gives bondholders fractional ownership of the firm. In our model, bondholders receive only a part of firm value as we assume that the rest is dissipated through inefficient management under bondholders. Thus, a key contribution of our model is to show that the option to acquire equity means that even straight debt shares the convertibility feature of convertible debt.

Our contention is that when firms have valuable growth options, liquidation value deviates from the value of the firm under bondholder control. Liquidation value is the depreciated value of assets in place. The value of the firm under bondholder control (also referred to as a debt for equity swap) is the sum of the value of assets in place and the value of growth options held by the firm. Bondholders may prefer to assume ownership rather than to liquidate if growth options are sufficiently valuable. It is this option to assume ownership that gives rise to the implicit warrant. The value of the warrant depends on the volatility of cash flows, being higher for firms with more volatile cash flows, *ceteris paribus*. It is in these very volatile firms with the correspondingly higher probability of distress that the put option sold by bondholders is worth more. The long position in the warrant offsets the short put option, potentially delivering lower yields on bonds issued by such firms.

Eom et al. (2004) compare pricing errors obtained from five different structural models of bond prices. They find that a common shortcoming of these models is they under-estimate

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yields on relatively safe corporate bonds and over-estimate yields on bonds issued by firms with high leverage and volatility. Our model predicts the largest reduction in yields for bonds issued by high risk firms with high growth potential. The warrant held by bondholders is most valuable in such firms. The value of the warrant declines in firms with very low growth potential, being the lowest in low risk firms with low growth potential.

Our paper belongs to a category of structural models that incorporate the debt for equity swap permitted under the re-organization clause of US Chapter 11 bankruptcy code. Mella-Barral and Perraudin (1997) and Anderson and Sundaresan (1996)) developed pricing models where bondholders are willing to negotiate debt service whenever liquidation value falls below the face value of debt. Bondholders are willing to negotiate in these models so as to avoid the deadweight costs associated with verification of liquidation value in the event of bankruptcy. These models are similar to ours in that bondholders have incentives to negotiate with equity holders, rather than to liquidate outright, a firm that is unable to meet its debt service. Where our models differ is that we allow growth options to affect bondholder incentives, including the incentive to negotiate debt service, and the incentive to take control of the firm in the event of bankruptcy.

Another category of models prices risky debt when absolute priority rules (APR) are violated in bankruptcy court. Longstaff and Schwartz (1995) derive a model of risky debt when APR violations lead to a lower payoff to bondholders in bankruptcy. Leland and Toft (1996) derive a model for the price of risky debt by allowing bankruptcy to be determined endogenously by shareholders. They also allow bondholders to recover less than their promised value, but do not explicitly identify APR violations as the reason for a lower payoff. Our model does not permit APR violations, but if such violations are common, it would reduce the value of the warrant, by reducing the proportion of firm value that bondholders receive in the event of a Chapter 11 re-organization. None of the other features of the model would be qualitatively affected by APR violations.

The rest of this paper is organized as follows. In section II, we describe the main assumptions underlying the model. In section III, we develop expressions for the market values of debt and equity. In Section IV, we examine the possibility of strategic debt service. In section V, we present our conclusions.

II. THE MODEL

A. Basic Assumptions

A firm has a collection of assets in place which generate a stochastic cash flow c_t given by the process:

$$dc_t = c_t(\mu_t dt + \sigma_t dW_t) \quad (1)$$

where drift μ and volatility σ are assumed to be constant. W_t is standard Brownian Motion. The risk-free rate of interest, r, is assumed to be constant. With risk neutrality, the value of assets in place, A_t, can be derived by solving the following equilibrium condition:

$$rA_t = c_t + d/d\Delta\, E_t(A_{t+\Delta})\,|_{\Delta=0} \quad (2)$$

If $A_t = A(c_t)$ is a twice continuously differentiable function of the state variable, c_t, by Ito's lemma, this function must satisfy the differential equation:

$$rA = c + \mu c\frac{\partial A}{\partial c} + \frac{1}{2}\sigma^2 c^2\frac{\partial^2 A}{\partial c^2} \quad (3)$$

We rewrite equation (3) as a stochastic differential equation (SDE):

$$0 = c + \mu c\frac{\partial A}{\partial c} + \frac{1}{2}\sigma^2 c^2\frac{\partial^2 A}{\partial c^2} - rA \quad (4)$$

The boundary conditions to be satisfied by asset value, A, are:

(1a) The no-bubbles condition requires that when c tends to infinity, A tends to $(c_t/(r-\mu))$. Evaluating the expectation operator:

$$E_t\left(\int_t^\infty c_v e^{-r(v-t)}\,dv\right) = \frac{c_t}{r-\mu} \quad (5)$$

(1b) No-arbitrage arguments imply that at the time of closure the value of assets is equal to the scrapping price, γ. Or, $A(c_q) = \gamma.c_q$ denotes the cash flow at which the firm would optimally close.

(1c) The smooth pasting condition, which requires that

$$\frac{\partial A}{\partial c_q} = 0$$

Proposition I: -*The following closed form expression is the equilibrium value of assets in place:

$$A(c) = \begin{cases} \dfrac{c}{r-\mu} + \left(\gamma - \dfrac{c_q}{r-\mu_t}\right)\left(\dfrac{c}{c_q}\right) & c \geq c_q \\ \gamma & c < c_q \end{cases} \quad (6)$$

Proof: See Appendix 1.

Figure 1 plots the value of assets in place for various values of the volatility parameter, σ. Parameter values are based on Mella-Barral and Perraudin (1997). The growth rate of cash flows, μ is assumed to be 2%, the risk-free rate of interest is 6% and liquidation value, γ, is chosen to be 2. Figure 1 shows that as long as cash flows are below bankruptcy level, the value of assets in place is equal to γ, the liquidation value. Asset value increases almost linearly with cash flow beyond the bankruptcy region. Figure 1 also plots the percent change in asset value at each cash flow level, for a change in the drift parameter from 1% to 2% ('vega with respect to drift'), and for a change in σ from 20% to 30% ('vega with respect to volatility'). The graph shows that an increase in drift leads to an increase in asset value that is dependent on the level of cash flows. At low cash flow levels, the value of assets in place is close to liquidation value. Hence, a change in the drift parameter leads to only a small increase in asset value. There is a larger increase in asset values when cash flows are higher.

The situation reverses when changes in σ are considered. There is a sharp initial increase in asset value. At higher cash flows, when asset value is comfortably above its scrapping value, an increase in σ leads to a smaller increase in asset value.

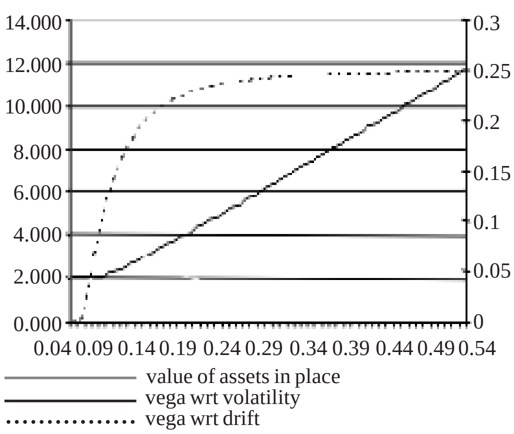

Fig. 1. Value of assets in place

B. The Growth Option

Each year the firm has a one-period option to invest in a new project. We assume that the investment is fully financed by reinvestment of cash flows generated by assets in place. Such an assumption may be justified by the pecking order theory of Myers and Majluf (1984), according to which firms favor internally generated funds as a source of capital to fund new investments. Future cash flows to the project are related to the level of investment and are given by:

$$g_{t+1} = (1 + \varepsilon_t)w(c_t - b) \quad (7)$$

where b is the level of debt service assumed to be constant. w is the plowback ratio assumed to be constant and ε_t is the stochastic return on assets which is assumed to follow a geometric Brownian motion given by:

$$d\varepsilon_t = \varepsilon_t(\beta dt + v dZ_t) \quad (8)$$

where drift β and volatility v are constant. Z_t is standard Brownian Motion and $dW_t dZ_t = \rho dt$. A constant plowback ratio implies an inefficient investment strategy on the part of shareholders when bankruptcy is imminent, but we retain this assumption here, and analyze shareholder incentives to invest in a later section.

We let $G(c_t, \varepsilon_t; b)$ be the time t value of this one-period growth option, which is governed by the following differential equation:

$$g_{t+1} - w(c-b) + \mu c\frac{\partial G}{\partial c} + \frac{1}{2}\sigma^2 c^2 \frac{\partial^2 G}{\partial c^2} + \beta\varepsilon\frac{\partial G}{\partial \varepsilon} + \frac{1}{2} v^2\varepsilon^2 \frac{\partial^2 G}{\partial \varepsilon^2} + \rho\sigma v c\varepsilon \frac{\partial^2 G}{\partial c \partial \varepsilon} - rG(c,\varepsilon) = 0 \quad (9)$$

In the equation above, cash flows from the growth option in equation (9) are net of the required investment, w (c_t-b). The boundary conditions that apply to the growth option are:

(2a) The no-bubbles condition requires that the value of the one-period option is the expected discount integral:

$$E_t\left[\int_t^\infty ((1 + \varepsilon_v)w(c_v - b) - w(c_v - b))e^{-r(v-t)}dv\right] =$$

$$\frac{(1 + \varepsilon_t)wc_t}{r - \beta - \mu - \sigma v \rho} - \frac{w(1 + \varepsilon_t)b \quad wc_t}{r - \beta} - \frac{wb}{r - \mu} + \frac{}{r}$$

(2b) Exercise of the growth option is triggered when cash flows and the ROA reach a critical level. Below these levels, $G(c_x, \varepsilon_x) = 0$

(2c) The smooth pasting condition (assuming $\varepsilon_x = \varepsilon$)

$$=> \frac{\partial G}{\partial c_x} = 0.$$

Proposition II: Let G denote the value of the one-period growth option. G has the following solution:

$$G(c,\varepsilon)= \begin{cases} \frac{w(1+\varepsilon_t)c}{r-\beta-\mu-\sigma\nu\rho}-\frac{w(1+\varepsilon)b}{r-\beta}-\frac{wc}{r-\mu}+\frac{wb}{r} \\ +\left(\frac{wc_x}{r-\mu}-\frac{wb}{r}-\frac{w(1+\varepsilon_x)c_x}{r-\beta-\mu-\sigma\nu\rho}+\frac{w(1+\varepsilon_x)b}{r-\beta}\right) \\ \left(\frac{c}{c_x}\right)^{\psi}\frac{(1+\varepsilon)}{(1+\varepsilon_x)} & c \geq c_x \\ 0 & c < c_x \end{cases}$$

.... (10)

c^{-}_{x} is the critical level of cash flow at which exercise of the growth option is triggered.

Proof: See Appendix 2.

Figure 2 is a graph of the growth option, G(C,ε) with parameter values set at ν= 5%, σ=20%, β= 1%. The growth option has the classic shape of a simple call option. The percent change in the value of the growth option for a change in n, the volatility of ROA, from 5% to 10% ("vega-ν), is also plotted for varying cash flow levels. An increase in ν leads to a large increase in the value of the growth option, the magnitude of which depends on the level of cash flows. At low cash flow levels, the probability of exercise of the growth option increases rapidly with ν. At higher cash flows, an increase in ν has a much smaller impact on the likelihood of exercise.

Figure 2 also plots the percent change in the value of the growth option for a change in σ from 20% to 30% (vega-σ). Vega-σ is lower than vega-ν at all cash flow levels. The level of reinvestment in the growth option and cash inflows into the growth option are simultaneously affected by an increase in σ. Higher volatility in cash flows has an adverse impact on the level of reinvestment in the growth option and a favorable impact on cash inflows into the investment project. The two countervailing effects lead to a positive, but smaller first derivative of the growth option with respect to σ. Other comparative statics are similar to those of a simple call option:

The expressions derived in this section for assets in place, and for the single-period growth option, will serve as building blocks for the derivation of the value of risky debt issued by the firm.

III. FIRM, EQUITY AND DEBT VALUES

Total firm value, $F(c_t, \varepsilon_t; b)$, is the sum of assets in place and the value of the growth option, $G(c_t, \varepsilon_t; b)$.

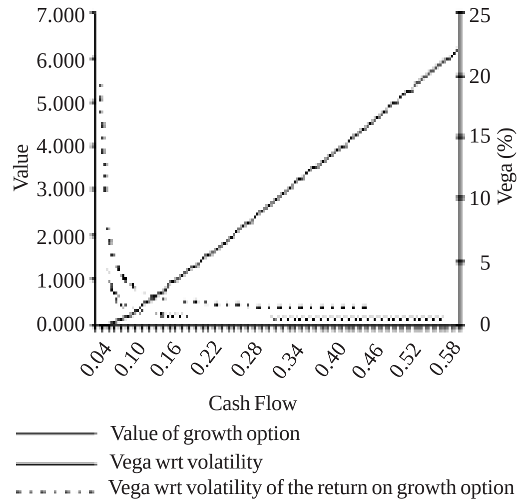

Fig. 2. Value of growth option

$$F(c_t, \varepsilon_t; b) = A(c_t)+G(c_t, \varepsilon_t; b) \text{ .... (11)}$$

Equation (11) with b=0, is also the value of equity if the firm is 100% equity financed.

III A. Equity Value

If $E_t = E(c_t, \varepsilon_t)$ is a twice continuously differentiable function of the state variables, c_t, and ε_t, by Ito's lemma, this function must satisfy the following partial differential equation (PDE) with mixed order term

$\frac{\partial^2 E}{\partial \varepsilon_t \partial c_t}$:

$$(1\text{-}w)(c\text{-}b)+\mu c\frac{\partial E}{\partial c}+\frac{1}{2}\sigma^2 c^2\frac{\partial^2 E}{\partial c^2}+\beta\varepsilon\frac{\partial E}{\partial \varepsilon}\nu^2\varepsilon^2\frac{\partial^2 E}{\partial \varepsilon^2}+\rho\sigma\nu c\varepsilon\frac{\partial^2 E}{\partial c\partial \varepsilon}$$
$$-rE(c,\varepsilon)=0 \text{ .... (12)}$$

We solve the PDE with the following boundary conditions:

(3a) The value of equity when c_t =>∞ is the expected discounted integral:

$$E_t\left[\int_t^{\infty}(1\text{-}w)(c_v\text{-}b)e^{-r(v-t)}dv\right]=\frac{(1\text{-}w)c_t}{r\text{-}\mu}-\frac{(1\text{-}w)b}{r}.$$

3b) When cash flows hit the bankruptcy level, c_b, equityholders as the residual claimants receive

$$E(c_b, \varepsilon_b) = \max\left[F(c_b,\varepsilon_b)-\frac{b}{r},0\right].$$

(3c) The smooth pasting condition, namely that

$\frac{\partial E}{\partial c_b} = 0$ (assuming $\varepsilon_b = \varepsilon$).

Proposition III: The value of equity in a levered firm is:

$$E(c,\varepsilon)=\begin{cases}\frac{(1\text{-}w)c}{r\text{-}\mu}-\frac{(1\text{-}w)b}{r}+\left(\frac{(1\text{-}w)b}{r}-\frac{(1\text{-}w)c_b}{r\text{-}\mu}\right)\left(\frac{c}{c_b}\right)^{\psi}\frac{\varepsilon}{\varepsilon_b} \\ \text{if } F(c_b,\varepsilon_b)\geq\frac{b}{r} \\ 0 \qquad \text{if } F(c_b,\varepsilon_b)<\frac{b}{r}\end{cases} \quad (13)$$

where F is the combined value of debt and equity and

$$c_b=-\frac{\psi b(r\text{-}\mu)}{(1\text{-}\psi)r},$$

is the cash flow level at which the firm declares bankruptcy

Proof: See Appendix 3.

III B. Value of Fixed-Rate Coupon Debt

Suppose the firm has issued a single homogenous class of perpetual debt with principal b/r and a coupon b, which is a constant perpetuity, unless the coupon is renegotiated in the event of financial distress. The assumption of perpetual debt is similar to Leland (1994) and may be justified if the firm issues fresh debt when existing debt reaches maturity. If agents are risk neutral, financial market equilibrium requires that L_t, the value of debt satisfies:

$$r\,L_t = b + d/d\Delta\, E_t\,(L_{t+\Delta})\,|_{\Delta=0} \quad (14)$$

If $L_t = L(\varepsilon_t, c_t)$ is a twice continuously differentiable function of the state variables, by Ito's lemma, this function must satisfy the differential equation:

$$b+\mu c\frac{\partial L}{\partial c}+\frac{1}{2}\sigma^2c^2\frac{\partial^2 L}{\partial c^2}+\beta\varepsilon\frac{\partial L}{\partial \varepsilon}+\frac{1}{2}\nu^2\varepsilon^2\frac{\partial^2 L}{\partial \varepsilon^2}+p\sigma\nu c\varepsilon\frac{\partial^2 L}{\partial c\partial\varepsilon}-rL(c,\varepsilon)=0 \quad (15)$$

Equation (15) is subject to the following boundary conditions:

(4a) When cash flows tend to infinity, the value of debt is the expected discounted integral,

$$E_t\left[\int_t^{\infty} be^{-r(v-t)}dv\right]=\frac{b}{r}.$$

(4b) As in Leland (1994), Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997), we assume that bankruptcy occurs when the firm cannot issue additional equity to service debt. Equity holders will choose to declare bankruptcy when cash flows from existing assets hit a trigger c_b, which is the solution to the smooth pasting condition, $E'(c_b) = 0$, where E is the value of equity.

In the event of bankruptcy, bondholders receive the face value of debt if liquidation value is higher than face value. If liquidation value is below face value, bondholders have the choice of liquidating the firm, or of transferring control of the firm to themselves. Mella-Barral and Perraudin (1997) model a similar choice faced by bondholders between liquidation or taking control. In their model, it is costly verification of liquidation value that gives the option to bondholders.

When control of the firm passes to bondholders, we assume that the firm is operated inefficiently. The value of assets-in-place drops to $\xi_0 A(c_t)$, and the value of the growth option drops to $\xi_1 G(c_t, \varepsilon_t)$ where ξ_0 and ξ_1 are both less than 1.0. The value of the firm in the hands of bondholders is therefore:

$$X(c_t,\varepsilon_t)=\xi_0 A(c_t)+\xi_1 G(c_t,\varepsilon_t;\, b=0) \quad (16)$$

The payoff to bondholders in the event of bankruptcy can now be expressed as:

$$L(c_b,\varepsilon_b)=\min\left[\frac{b}{r},\ \max\,[X(c_b,\varepsilon_b),\,A(c_b)]\right] \quad (17)$$

By a simple manipulation, equation (17) may be expressed as a combination of a long call option and a short put option:

$$L(c_b,\varepsilon_b)=\max(X\text{-}A(c_b),0)-\max(b/r-A(c_b),0) \quad (18)$$

The put option is on the liquidation value of the firm with the face value of debt as the exercise price. The call option is on the re-organization value of the firm with liquidation value as the exercise price. Thus, bondholders hold a warrant to convert their debt into equity. In this respect, straight debt is like a convertible bond. Where the two types of debt differ are the regions of cash flows where convertibility has value; for convertible bonds, the convertibility option is in-the-money in up- states, whereas the call option held by straight debt is in-the-money in down-states.

Proposition IV: The ODE in equation (15) can be solved subject to boundary conditions (4a) and (4b) yielding,

$$L(c,\varepsilon)=\begin{cases}\frac{b}{r}+\left[l(cb,eb)-\frac{b}{r}\right]\left[\frac{c}{c_b}\right]^{\psi}\frac{\varepsilon}{\varepsilon_b} & \text{if } l(cb,eb)<\frac{b}{r} \\ \frac{b}{r} & \text{if } l(cb,eb)\geq\frac{b}{r}\end{cases} \quad (19)$$

where $l(c_b,e_b) = \max\,[X(c_b,\varepsilon_b),\,A(c_b)]$ is the value of debt in bankruptcy.

Proof: See Appendix 4.

Equation (19) can easily be interpreted as the expected value of the payoff to bondholders, $L(c,\varepsilon) = [1-\pi_b]\, b/r + \pi_b\, -l(c_b,\varepsilon_b)$ where

$$\Pi_b = \left(\frac{c}{c_b}\right)^{\psi} \frac{\varepsilon}{\varepsilon_b}$$

is the risk-neutral probability that cash flows fall below the bankruptcy level

Figure 3 plots the values of debt at various cash flow levels. At low cash flow levels, the value of debt is constant at the liquidation value of the firm. At higher cash flow levels, bondholders become owners of the firm and the value of debt increases sharply with cash flows. At even higher cash flow levels, the value of debt asymptotically approaches the face value, b/r.

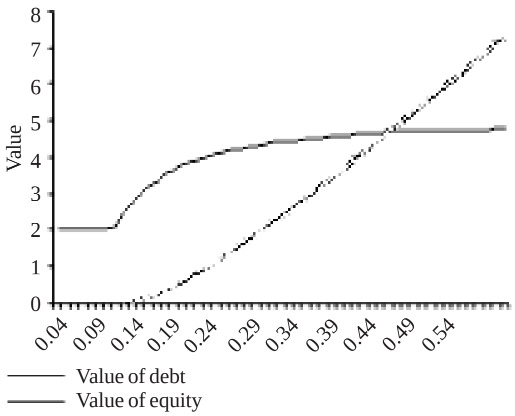

Fig. 3. Values of debt and equity

Figure 3 also has a plot of equity value. The value of equity is zero when the firm is bankrupt because we assume that priority rules are upheld in bankruptcy court. Beyond the bankruptcy region, there is a range of cash flows over which the relation between value of equity and cash flows is convex. In this region of cash flows, any increase in firm value due to an increase in cash flows is shared with bondholders up to the point where there is no longer a threat of bankruptcy. Beyond this region, debt becomes risk-free and shareholders are the only beneficiaries from an increase in firm value.

In Figure 4, we plot the sensitivity of debt to a change in parameter values. The parameters are the volatility of cash flows,σ, and ν, the volatility of the return on investment in the growth option. The graph plots the change in the value of debt, 'vega- sigma', for an increase in σ from 20% to 30%. At very low cash flow levels, the vega of debt is zero reflecting the fact that debt value is the constant liquidation value. Thereafter, in the region of cash flows up to c_b, value of debt is sensitive to an increase in σ as bondholders control the firm. Debt value increases in this region because debtors hold a warrant on the firm, whose value increases with σ. At cash flow levels beyond bankruptcy, the value of debt decreases with an increase in σ. The decrease occurs because bondholders hold a short position in a put option, the value of which decreases with an increase in σ.

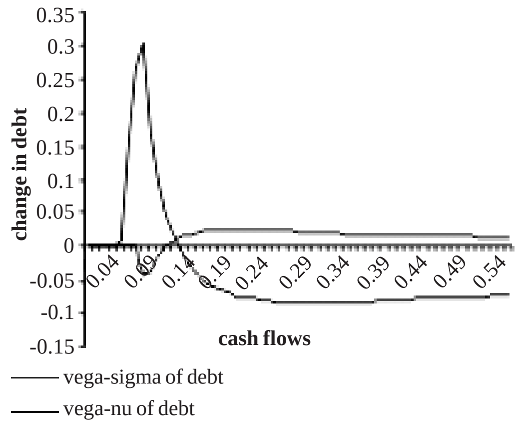

Fig. 4. Sensitivity of debt to a change in parameter values

The figure also plots the change in the value of debt as ν, 'vega-nu', increases from 5% to 10%. The figure shows that 'vega-nu' is the inverse of 'vega'. At cash flow levels that lead to liquidation of the firm, 'vega-–ν' is zero. At slightly higher cash flows, when bondholders take control, the value of debt decreases with ν. The growth option is a short-maturity deep in the money option when bondholders take control. The increase in ν reduces the value of the growth option by increasing the likelihood that the option is out of the money. Thus, unlike an increase in cash flow volatility, which could be beneficial for the value of debt in some regions of cash flows, an increase in the volatility of the return on investment on the growth option has a uniformly deleterious effect on the value of debt.

The analysis in this section shows that in the absence of the warrant, the value of debt would be lower as the firm is liquidated as soon as cash flows fall below the level required for debt service. It is the presence of the warrant that lowers bond yields in firms with volatile cash flows. Eom et al. (2004) find that a common shortcoming of a number of structural models of bond prices is that they under-estimate yields on relatively safe corporate bonds and over-estimate yields on bonds issued by firms with high leverage and volatility. Our model does not suffer from this shortcoming, as it predicts that the value of the warrant held by bondholders is higher for high-risk (volatile cash flows) firms that have valuable growth options. The value of the warrant decreases as the value of growth options decreases. Thus, among high risk firms, those with no valuable growth options are predicted to carry the highest bond yields.

IV. STRATEGIC DEBT SERVICE

Myers (1977) shows that the presence of risky debt leads to underinvestment in positive NPV projects. When debt is risk-free, shareholders invest in all positive NPV projects. When debt becomes risky, there is a transfer of wealth to bondholders when shareholders contribute equity capital to invest in growth options. Under-investment in the growth option is a usual outcome. Strategic debt service may be a remedy to the acute under-investment problem in a firm facing financial distress. Several papers (Anderson and Sundaresan 1996; Mella-Barral and Perraudin 1997) incorporate strategic debt service into their models of corporate bond prices. Bondholders are willing to re-negotiate the terms of their claim in these models to avoid a costly resolution of financial distress. In our model, bondholders have greater bargaining power in states where the value of assets in place is low, but the growth option has a high NPV. Bondholders can force the firm to declare bankruptcy if the value of assets in place deteriorates to a level equal to the face value of debt, and exercise their option to take control of the firm. Taking over control is preferred to negotiation of the terms of the debt offering when the option to convert is deep in the money. Costly resolution of financial distress, which is admittedly ignored in this model, in fact strengthens bondholders' bargaining power by increasing their incentive to swap their debt for equity, rather than to liquidate.

Bondholders can be persuaded to re-negotiate the terms of their debt only if the new coupon is set at a level at which the face value of new debt is equal to the maximum of liquidation value or value of the firm under bondholder control. That is:

$$s/r = \max(\gamma, X) \quad (25)$$

If the firm has no growth options, and X differs from γ due to deadweight costs only, the negotiated coupon, s would be set equal to:

$$s = \frac{cr}{r-\mu} + \left(\gamma - \frac{c_q}{r-\mu_t}\right)\left(\frac{cr}{c_q}\right)^{\lambda} \quad (26)$$

The negotiated coupon, s, is higher in the presence of growth options. Bondholders would be willing to forego the warrant held by them to convert their debt to equity, which is more valuable when growth options are deep in the money, only at higher coupon levels. The negotiated coupon can be solved by setting:

$$s/r = \max(\gamma, \xi_0 A + \xi_1 G) \quad (27)$$

The magnitudes of the parameters ξ_0 and ξ_1 determine whether bondholders capture rents from shareholders in the negotiations to restructure debt.

The comparative statics of the renegotiated coupon, s, are identical to those of A, the assets in place, and those of G, the value of growth options. The magnitude of s increases with G and A. Since these asset values are positively related to σ, the volatility of cash flows, it follows that the magnitude of s, the negotiated coupon, also increases with σ. Higher renegotiated coupons make it less likely that debt issued by firms with higher volatility is renegotiated. The decrease in the probability of renegotiation and the ability of bondholders to capture rents from shareholders in the negotiating process implies a higher value for debt issued by riskier firms. In the earlier section we showed that bondholders in risky firms benefit from higher values for the warrant. This section shows that in addition, bondholders in risky firms benefit from their ability to extract rents from shareholders during debt service negotiations.

V. CONCLUSION

We derive a closed-form expression for the value of risky debt when firms have a simple one-period growth option to invest in a project. In the presence of growth options, bondholders in financially distressed firms hold an option to liquidate the firm, or to take control of the firm. Our paper is the first to incorporate these options held by bondholders into a structural model of corporate bond prices. We show that the advantage of our model is that it implies higher prices for bonds issued by volatile firms with more growth opportunities than do existing structural models of risky debt. Two features in our model contribute to the higher price. Bondholders have a warrant to convert their debt to equity when the firm lands in bankruptcy, the value of which is higher for firms with more valuable growth options. Growth options also enable bondholders to have greater bargaining power in resisting shareholders' attempts to renegotiate the terms of debt when financial distress is imminent.

This line of research can be extended to investigate the impact of growth options on optimal capital structure. Our model suggests that relatively higher prices for bonds in high growth firms imply that bankruptcy costs are lower than is commonly assumed. Debt capacity should increase in response to the lower distress risk. These issues are beyond the scope of this paper and are left to future research.

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NOTES

1. Gilson, John and Lang (1990) find that a roughly equal proportion of firms in financial distress restructure their debt as liquidate. James (1995) finds that the structure of public and private debt claims affects the success of public debt exchange offers of firms in financial distress. Hotchkiss (1995) and Gilson (1997) study how debt restructuring of financially distressed firms affects subsequent performance of these firms.

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APPENDIX 1

Proof of Proposition 1

The SDE in equation (4) has the following solution (for the sake of no-explosion, we set one of the integration constants to zero):

$$A(c) = n_1 c^{\lambda} \quad \text{(A1)}$$

where λ is the negative root of the quadratic equation:

$$\frac{1}{2}\sigma^2 \lambda(\lambda - 1) + \lambda\mu - r = 0$$

Applying boundary conditions (1a) and (1b) in the main text, to equation (A1), we get:

$$A(c_q) = \frac{c_q}{r\text{-}\mu} + n_1 c_q^{\lambda} = \gamma$$

$$\text{N1} = \left(\gamma - \frac{c_q}{r\text{-}\mu}\right) c_q^{-\lambda_1} \quad \text{(A2)}$$

Substituting back into equation (A1) yields:

$$A(c) = \frac{c}{r\text{-}\mu} + \left(\gamma - \frac{c_q}{r\text{-}\mu_t}\right)\left(\frac{c}{c_q}\right)^{\lambda} \quad \text{(A3)}$$

The bankruptcy trigger point c_q is obtained by applying the third boundary condition to equation (A3).

$$\frac{\partial A}{\partial c_q} = \left(\frac{c}{c_q}\right)^{\lambda}\left(-\lambda\gamma c_q^{-1} - (1\text{-}\lambda)\frac{1}{r\text{-}\mu}\right) = 0$$

$$c_q = \frac{-\lambda\gamma(r\text{-}\mu)}{(1\text{-}\lambda)} \quad \text{(A4)}$$

Substituting for c_q into equation (A3) leads to the result.

APPENDIX 2

Proof of Proposition 2

The SDE in equation (9) has the following solution (for the sake of no-explosion, we set one of the integration constants to zero):

$$G(c,\varepsilon) = N_2 c^{\psi} (1+\varepsilon) \qquad \text{(B1)}$$

where ψ is the negative root of the quadratic equation:

$$\frac{1}{2}\sigma^2\psi(\psi - 1) + \psi(\mu + \sigma\nu\rho) + (\beta - r) = 0$$

Applying boundary conditions (2a) and (2b), we get:

$$G(c_x,\varepsilon_x) = \frac{w(1+\varepsilon_x)\,c_x}{r-\beta-\mu-\sigma\nu\rho} - \frac{w(1+\varepsilon_x)\,b}{r-\beta} - \frac{wc_x}{r-\mu} + \frac{wb}{r} + N_2 c_x^{\psi}(1+\varepsilon) = 0$$

$$N_2 = \left[\frac{wc_x}{r-\mu} - \frac{wb}{r} - \frac{w(1+\varepsilon_x)\,c_x}{r-\beta-\mu-\sigma\nu\rho} + \frac{w(1+\varepsilon_x)\,b}{r-\beta}\right] c_x^{-\psi}(1+\varepsilon_x)^{-1} \qquad \text{(B2)}$$

Thus we get:

$$G(c,\varepsilon) = \frac{w(1+\varepsilon)\,c}{r-\beta-\mu-\sigma\nu\rho} - \frac{w(1+\varepsilon)\,b}{r-\beta} - \frac{wc}{r-\mu} + \frac{wb}{r} + \left[\frac{wc_x}{r-\mu} - \frac{wb}{r} - \frac{w(1+\varepsilon_x)\,c_x}{r-\beta-\mu-\sigma\nu\rho} + \frac{w(1+\varepsilon_x)\,b}{r-\beta}\right]\left(\frac{c}{c_x}\right)^{\psi}\frac{(1+\varepsilon)}{(1+\varepsilon_x)} \qquad \text{(B3)}$$

Applying condition (2c), we solve for exercise trigger C_x:

$$\frac{\partial G}{\partial c_x} = \left(\frac{c}{c_x}\right)^{\psi}\frac{\varepsilon}{\varepsilon_x}\left[\psi c_x^{-1}\left(\frac{b}{r} - \frac{w(1+\varepsilon_x)\,b}{r-\beta}\right) + (1-\psi)\left(\frac{w(1+\varepsilon_x)}{r-\beta-\mu-\sigma\nu\rho} - \frac{1}{r-\mu}\right)\right] = 0$$

$$\Rightarrow c_x = \frac{\psi\left(\frac{b}{r} - \frac{w(1+\varepsilon_x)\,b}{r-\beta}\right)}{(1-\psi)\left(\frac{w(1+\varepsilon_x)}{r-\beta-\mu-\sigma\nu\rho} - \frac{1}{r-\mu}\right)} \qquad \text{(B4)}$$

Substituting for c_x back into equation (B1) yields the final solution.

APPENDIX 3

The SDE in equation (12) has the following solution (for the sake of no-explosion, we set one of the integration constants to zero):

$$E(c,\varepsilon) = N_3 c^{\psi}\varepsilon \qquad \text{(C1)}$$

where ψ has already been defined under the growth option derivation.

Applying boundary conditions (3a) and (3b), we see that if

$$F(c_b, \varepsilon_b) < \frac{b}{r}:$$

$$E(c_b, \varepsilon_b) = \frac{(1-w)c_b}{r-\mu} - \frac{(1-w)b}{r} + N_3 c_b^{\psi}\varepsilon = 0$$

$$N3 = \left[\frac{(1-w)b}{r} - \frac{(1-w)c_b}{r-\mu}\right] c_b^{-\psi}\varepsilon_b^{-1}$$

Thus we get:

$$E(c,\varepsilon) = \frac{(1-w)c}{r-\mu} - \frac{(1-w)b}{r} + \left[\frac{(1-w)b}{r} - \frac{(1-w)c_b}{r-\mu}\right]\left(\frac{c}{c_b}\right)^{\psi}\frac{\varepsilon}{\varepsilon_b} \qquad \text{(C2)}$$

Bankruptcy trigger c_b can be solved by applying condition (3c) to equation (C2):

$$\frac{\partial E}{\partial c_b} = \left(\frac{c}{c_b}\right)^{\psi}\frac{\varepsilon}{\varepsilon_b}\left[-(1-\psi)\frac{(1-w)}{r-\mu} - \psi c_b^{-1}\frac{(1-w)b}{r}\right] = 0$$

$$c_b = -\frac{\psi b(r-\mu)}{(1-\psi)r} \qquad \text{(C3)}$$

APPENDIX 4

With boundary conditions (4a) and (4b), the ODE in equation (15) has the following solution (for the sake of no-explosion, we set one of the integration constants to zero):

$$L(c,\varepsilon) = \frac{b}{r} + N_4 c^{\psi}\varepsilon \qquad \text{(D1)}$$

where ψ has been defined under the growth option derivation. If

$$C(c_b, \varepsilon_b) < \frac{b}{r};$$

$$L(c_b, \varepsilon_b) = \frac{b}{r} + N_4 c_b^{\psi}\varepsilon = C(c_b, \varepsilon_b)$$

$$N_4 = \left[C(c_b, \varepsilon_b) - \frac{b}{r}\right] c_b^{-\psi}\varepsilon_b^{-1} \qquad \text{(D2)}$$

Thus,

$$L(c,\varepsilon) = \frac{b}{r} + \left(C(c_b, \varepsilon_b) - \frac{b}{r}\right)\left(\frac{c}{c_b}\right)^{\psi}\frac{\varepsilon}{\varepsilon_b} \qquad \text{(D3)}$$